Comment on physical-scalar mediated contribution to fermion self energy

Biswajit Adhikary*
Saha Institute of Nuclear Physics,
1/AF Bidhan Nagar, Kolkata 700064, India

Abstract

We calculate the physical scalar contribution to the fermionic self energy matrix at one loop. We make a comment about the difference of our results from those in the existing literature.

We want to calculate physical scalar contribution to the fermionic self energy matrix in a generalized gauge theory that was originally done by Weinberg [1]. In general there should be a symmetry breaking scalar potential. Non zero vacuum expectation values of the scalar fields break the gauge symmetry to lower group and generate mass terms for gauge fields and fermionic fields. The scalar potential about the vacuum generate mass term for scalar fields also. The scalar mass matrix have nonzero eigenvalues along with the zero eigenvalues. Scalar fields with zero mass are known as Goldstone scalars which can be absorbed into the gauge fields under proper gauge transformation. Rest of the scalars are physical. We have chosen the basis of the scalar fields where they have definite mass. We only concentrate on the contribution of those physical scalars to fermionic self energy matrix at the 1-loop level. The relevant part of the Lagrangian for our calculations is

$$\mathscr{L} = -\bar{f}_a m_{ab} f_b - \sum_i \bar{f}_a (\Gamma_i)_{ab} f_b h_i \tag{1}$$

where f_a's, h_i's, m and Γ_i's are respectively the fermionic fields, physical scalar fields with definite mass M_i's, zeroth order fermionic mass matrix and Yukawa coupling matrices of the fermions with the scalars h_i's. The Feynman diagram relevant for this calculation is in Figure. 1. We choose a basis of the fermionic fields where the fermionic mass matrix is free from γ_5. So, it will be hermitian. The fermionic fields in this basis will be $\hat{f} = Sf$ where S is a unitary

*biswajit.adhikari@saha.ac.in

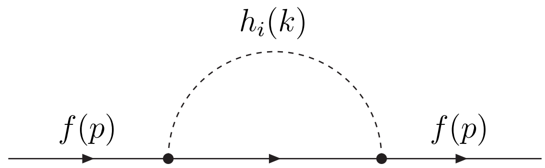

Figure 1: Self energy diagram of fermion with physical scalar.

transformation matrix. The fermionic mass matrix, Yukawa coupling matrix and self energy matrix in this basis will be [2]

$$\hat{m} = \gamma_0 S \gamma_0 m S^\dagger, \quad \hat{\Gamma}_i = \gamma_0 S \gamma_0 \Gamma_i S^\dagger \quad \text{and} \quad \hat{\Sigma} = \gamma_0 S \gamma_0 \Sigma S^\dagger \tag{2}$$

where Σ is fermionic self energy matrix in the original basis. So, the physical scalar contributions to the self energy matrix in this basis will be

$$i\hat{\Sigma}^{\text{scal}} = \sum_i \int \frac{d^4k}{(2\pi)^4} (-i\hat{\Gamma}_i) \times \frac{i}{k^2 - M_i^2} \times \frac{i}{\not{p} - \not{k} - \hat{m}} \times (-i\hat{\Gamma}_i) \tag{3}$$

The above Eq. (3) can be written down as

$$i\hat{\Sigma}^{\text{scal}} = \sum_i \int \frac{d^4k}{(2\pi)^4} \hat{\Gamma}_i \times \frac{1}{k^2 - M_i^2} \times \frac{\not{p} - \not{k} + \hat{m}}{(p-k)^2 - \hat{m}^2} \times \hat{\Gamma}_i. \tag{4}$$

There are basically two fundamental integrals. Their forms in d dimension for dimensional regularization scheme are

$$\begin{aligned} I &= \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2 - M_i^2} \times \frac{1}{(p-k)^2 - \hat{m}^2} \\ I^\mu &= \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2 - M_i^2} \times \frac{k^\mu}{(p-k)^2 - \hat{m}^2}. \end{aligned} \tag{5}$$

Results of the integrals under Feynman parameterization are

$$\begin{aligned} I &= \frac{i}{(4\pi)^{d/2}} \Gamma(\frac{4-d}{2}) \int_0^1 dx \mathscr{D}^{-(4-d)/2} \\ I^\mu &= p^\mu \frac{i}{(4\pi)^{d/2}} \Gamma(\frac{4-d}{2}) \int_0^1 x dx \mathscr{D}^{-(4-d)/2} \end{aligned} \tag{6}$$

where

$$\mathscr{D} = \hat{m}^2 x + M_i^2 (1-x) - p^2 x (1-x). \tag{7}$$

Using the integral results of Eq. (6) in the Eq. (4) we have

$$\begin{aligned} i\hat{\Sigma}^{\text{scal}} &= \sum_i (\mu^{(4-d)/2} \hat{\Gamma}_i) \times \{(\not{p} + \hat{m}) I - \gamma_\mu I^\mu\} \times (\mu^{(4-d)/2} \hat{\Gamma}_i) \\ &= \sum_i (\mu^2)^{(4-d)/2} \hat{\Gamma}_i \frac{i}{(4\pi)^{d/2}} \Gamma(\frac{4-d}{2}) \int_0^1 dx \{\not{p}(1-x) + \hat{m}\} \mathscr{D}^{-(4-d)/2} \hat{\Gamma}_i. \end{aligned} \tag{8}$$

where μ is an arbitrary mass scale which has been introduced to keep Γ_i dimensionless in d dimension. Now using the expansion of type $A^{\epsilon/2} = 1 + \frac{\epsilon}{2}\ln(A) + O(\epsilon^2)$ and $\Gamma(\frac{\epsilon}{2}) = \frac{2}{\epsilon} - \gamma + O(\epsilon)$ where $\epsilon = 4 - d$, we obtain

$$\begin{aligned} i\hat{\Sigma}^{\rm scal} &= \sum_i \hat{\Gamma}_i \frac{i}{(4\pi)^2}\Big[W\int_0^1 dx\{\not{p}(1-x)+\hat{m}\} \\ &\quad - \int_0^1 dx\{\not{p}(1-x)+\hat{m}\}\ln\mathscr{D} + O(\epsilon)\Big]\hat{\Gamma}_i. \end{aligned} \tag{9}$$

where

$$W = \ln(4\pi) + \frac{2}{\epsilon} - \gamma + \ln(\mu^2). \tag{10}$$

Due to the scalar pseudoscalar bi-linear combination of Yukawa term, $\hat{\Gamma}_i\not{p} = \not{p}\gamma_0\hat{\Gamma}_i\gamma_0$. The self energy matrix can be written as

$$\Sigma = (\not{p} - \hat{m})F(p^2) + G(p^2)(\not{p} - \hat{m}) + \Sigma_{\rm eff}(p^2) \tag{11}$$

Upto first order term in F, G and $\Sigma_{\rm eff}$ fermionic propagator can be written down as [1]

$$S_F(p) = \frac{1}{1+G} \times \frac{1}{\not{p} - \hat{m} + \Sigma_{\rm eff}} \times \frac{1}{1+F}. \tag{12}$$

It shows that the pole of the propagator does not depend on F and G. So, we can easily substitute $\not{p}$ by $\hat{m}$ whenever $\not{p}$ will appear at the extreme right or extreme left in the expression for Σ. Following the above discussion and using $p^2 = \hat{m}^2$ inside $\mathscr{D}$ under the consideration of first order correction of mass we obtain

$$\begin{aligned} \hat{\Sigma}^{\rm scal} &= \sum_i \frac{1}{(4\pi)^2}\Big[W\int_0^1 dx\{\hat{m}\gamma_0\hat{\Gamma}_i\gamma_0(1-x) + \hat{\Gamma}_i\hat{m}\} \\ &\quad - \int_0^1 dx\{\hat{m}\gamma_0\hat{\Gamma}_i\gamma_0(1-x) + \hat{\Gamma}_i\hat{m}\}\ln\{\hat{m}^2x^2 + M_i^2(1-x)\} + O(\epsilon)\Big]\hat{\Gamma}_i. \end{aligned} \tag{13}$$

Coefficient of W is equivalent to the coefficient of $\ln(\Lambda^2)$ in [1] where cutoff regularization was used. Compared to the expression of [1] this result has different signs in the terms containing the combination $\gamma_0\hat{\Gamma}_i\gamma_0$, both in finite as well as diverging parts. Later various people [2, 3] used the results of Weinberg [1]. Turning back to the original basis with the relations in Eq. (2), using the hermiticity of $\hat{m}$ and the decompositions like $B = B_R(1+\gamma_5)/2 + B_R^\dagger(1-\gamma_5)/2$ for both m and Γ_i due to the hermiticity of the Lagrangian we obtain

$$\begin{aligned} \Sigma^{\rm scal} &= \sum_i \frac{1}{(4\pi)^2}\Big[W\int_0^1 dx\{m\Gamma_i^\dagger(1-x) + \Gamma_i m^\dagger\} \\ &\quad - \int_0^1 dx\{m\Gamma_i^\dagger(1-x) + \Gamma_i m^\dagger\}\ln\{mm^\dagger x^2 + M_i^2(1-x)\} + O(\epsilon)\Big]\Gamma_i. \end{aligned} \tag{14}$$

which is similar to the results of [2] except the sign correction here.

Note Added: After doing the above calculations we came to know that in another paper [4] the authors used the results of [1]. Later [5] it was commented that there was a sign error in [4].

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